

Comments on the “New VIX”

The CBOE began trading cash-settled options based on the VIX on February 24, 2006. VIX traders should be familiar with its calculation methodology. In this article, the authors answer the following questions: How large an impact is there on the VIX when an option goes into or out of the calculation? Do certain options impact the calculation of the VIX more than others? What are the implications for the settlement of derivative contracts based on VIX?

General

The CBOE has calculated and disseminated the widely watched VIX Index continuously since 1993, changing the calculation methodology in September 2003. The index represents a measure 30 day volatility implied by the prices of S&P500 (SPX) index options. The CBOE also publishes similar indexes, including: VXD, which is based on Dow Jones Industrial Average (DJX) options; VXN, which is based on NASDAQ 100 Index (NDX) options; and RVX, which is based on Russell 2000 Index (RUT) options. In what follows, we will refer to the VIX; similar comments are also applicable to the VXD, VXN and RVX.

The VIX is calculated four times per minute (every fifteen seconds) using the quoted bid and offer of all options on the index from two consecutive expiration months, which have a non-zero bid price. This is quite different than the original VIX which was calculated only using the implied volatilities of the at the money (ATM) options. The exact procedures and formulas for calculating the VIX are given in the “VIX White Paper” which is available from the CBOE (www.cboe.com/micro/vix/vixwhite.pdf).

One aspect of the index calculation is particularly noteworthy. The VIX calculation procedure is to start with the ATM option and look at higher strike calls. We include all the calls for which we have a price, until we encounter two consecutive call options for which the bid price is zero. Similarly, on the downside, we include put prices until we encounter two consecutive put options whose bid price is zero. Since the current bid price is used to determine whether or not a constituent option is included in the VIX calculation the number of options used can

change from one moment to the next, as bid prices are changed.

There are currently cash-settled futures contracts based on VIX, which are traded on the CBOE Futures Exchange (CFE). The CBOE began trading cash-settled options based on the VIX on February 24, 2006. Any market participant trading VIX futures or VIX options should be familiar with the methodology used to calculate the VIX. They should also understand the practical aspects of the trading and settlement

- What are the implications for the settlement of derivative contracts based on VIX?

Discussion

Let's examine the VIX calculation in more detail. As shown in the following formula, a particular option impacts the VIX through a relative weighting scheme that includes the interval between strike prices and is also inversely proportional to the strike of the option.

From the formula it can be seen that the weight given to a particular option premium is determined by $\Delta K/K^2$ where K is the strike of the particular option and ΔK is the interval between strike prices. Therefore, for an equivalent price change, a lower strike option has a higher impact on the VIX quote. Similarly, price changes for options where there is a larger interval

“The number of options used can change from one moment to the next.”

of derivatives contracts based on VIX. We intend to cover these issues in a series of papers of which this is the first one.

Questions

- How large an impact is there on the VIX when an option goes into or out of the calculation?
- Do certain options impact the calculation of the VIX more than others?

between available strike prices also have a relatively larger impact on the VIX. From this, it follows that the options with the highest impact on the quote of the VIX for a given price change are typically low strike puts.

Consider the following example. On June 12, 2006, the calculated VIX index level using the midpoint of quoted bid offer spread was 18.963. The following

$$\sigma_{VIX}^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2$$

| | |
|----------------|------------------------------------------------------------|
| σ_{VIX} | The new VIX level |
| T | Time to expiration |
| F | Forward index level |
| K_i | Strike price of i^{th} option |
| ΔK_i | Interval between strike prices |
| K_0 | First strike below the forward index level, F |
| R | Risk-free interest rate to expiration |
| $Q(K_i)$ | The midpoint of the quoted spreads for the i^{th} option |

| Data used in VIX Calculation | | | | | Effect of change in midpoint | |
|------------------------------|----------|------|-------|----------|------------------------------|---------------|
| Strike | Put/Call | Bid | Offer | Midpoint | New VIX | Change in VIX |
| 950 | put | 0.1 | 0.6 | 0.35 | 18.986 | 0.023 |
| 1000 | put | 0.5 | 1 | 0.75 | 18.984 | 0.021 |
| 1050 | put | 1.3 | 1.8 | 1.55 | 18.977 | 0.014 |
| 1235 | put | 19.5 | 21.1 | 20.3 | 18.964 | 0.001 |
| 1240 | put | 20.9 | 22.9 | 21.9 | 18.964 | 0.001 |
| 1245 | put | 22.5 | 24.5 | 23.5 | 18.964 | 0.001 |
| 1350 | call | 0.4 | 0.55 | 0.475 | 18.964 | 0.001 |
| 1355 | call | 0.25 | 0.5 | 0.375 | 18.964 | 0.001 |
| 1360 | call | 0.25 | 0.35 | 0.3 | 18.964 | 0.001 |

table shows the data used to calculate the VIX for nine of the 88 options used to calculate the index. The table includes the three lowest strike puts, three puts nearest the at-the-money strike price, and the three highest strike calls. It also shows the effect of a 0.05 change in the midpoint used for each option to calculate the index.

As expected, the lowest strike option (950 put) has the greatest impact on the VIX level, over 23 times the impact for the same price change in the near-the-money or highest strike options. Among low strike puts, options where the bid price is zero have the greatest potential to impact the VIX. How much can the index level change when a previously no-bid put option is changed to a \$0.05 bid and thus is added to the VIX? Consider the previous calculation of the VIX. Three put options were excluded from the calculation for having a zero bid, including the 800, 850 and 900 Put. The following table illustrates the effect of changing the bid on either the 850 or 900 Put from zero to 0.05. Assuming all other option prices have remained unchanged, changing the 850 bid to 0.05 changes the VIX by 0.373. Most of this

change results from the relatively wide interval between the 850 and 950 put options used in the calculation. If for instance, the bid in both the 850 and 900 puts were changed to 0.05 the effect on the VIX would be less. Of course, this is just one example. In various situations, these changes could be higher or lower.

Finally, consider that options from two different expiration dates are used in the VIX calculation. Since the methodology interpolates to determine a constant 30-day measure of the VIX, options whose expiration is closer to 30 days will have a larger impact.

Settlement of derivative contracts on VIX

We have seen that the number of options used in the VIX calculation can have a noticeable impact on the VIX, the lowest strike puts have the greatest impact on the VIX level for a given price change, and that options whose expiration is closer to 30 days matter most. What does this mean for the settlement of a derivative contract based on the VIX?

The CBOE has taken steps to ensure that the option prices used in determination of the settlement value are fair.

These steps include, creating a special opening quotation of the VIX Index for settlement, instituting a special opening procedure for the series used in the settlement of a VIX contract, and making the VIX settlement date 30 days prior to the next options expiration.

Much of the discussion thus far has concerned the calculation of the VIX index as it is performed on a daily basis. Only quoted prices of the options determine the quote for the VIX Index. Most cash-settled derivative contracts are settled based on traded prices as opposed to quoted prices. Similarly, derivative contracts based on the VIX settle to a special opening quotation of the VIX, which may include both the traded prices for options and quotes if a trade does not occur. Traded prices are used only to the extent that the option continues to be bid after the opening trade has occurred.

An opening procedure was implemented that requires that orders in SPX options related to an expiring VIX position be entered prior to 8:00 am Chicago time. After 8:00, CBOE publishes imbalances and the projected opening price in each of the options used to calculate VIX. From 8:00 to 8:28 a.m., CBOE

| Actual Quote | | | | | Effect of change in midpoint | |
|--------------------------------|----------|-----|-------|----------|------------------------------|---------------|
| Strike | Put/Call | Bid | Offer | Midpoint | New VIX | Change in VIX |
| 850 | put | 0 | 0.5 | 0 | 19.336 | 0.373 |
| 900 | put | 0 | 0.2 | 0 | 19.028 | 0.065 |
| 0.05 Bid in both series | | | | | 19.171 | 0.208 |

opens the SPX electronic order book to all market participants in order to maximize liquidity in the SPX series used to calculate the VIX settlement.

In the case of VIX futures and options, the most important benefit of

using traded prices is the ability to trade SPX options at the prices used to calculate the settlement value. This ability to trade the settlement value results in enhanced liquidity for VIX futures and options. However, it may also result in a

disparity between the Final Settlement Value of an expiring volatility index futures or options contract and the opening indicative volatility index value on the final settlement date. This will be the subject of a future paper on VIX.



John Hiatt, CFA, Director of Research, Chicago Board Options Exchange
Contact: hiattj@cboe.com / (312)786-7779

Izzy Nelken, President, Super Computer Consulting, Inc.
Contact: izzy@supercc.com / (847)562-0600