Weather Derivatives – Pricing and Hedging

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Version updated : January 3, 2000

With thanks to Dan Mainka.

Introduction

The weather derivatives market has never been hotter. It is estimated that some 1600 deals worth approximately $3.5 billion have been done in the USA. Recently, the Chicago Mercantile Exchange (CME) has begun to list these products. In addition, there are several weather derivatives brokers who assist in closing over the counter (OTC) deals. These include: Natsource, EuroBrokers, Sakura Dellsher Inc., United Energy, Tradition and others.

Description of the derivative

Most weather derivatives have payouts which are based on "degree days". The contract is best illustrated by an example:

Type of option: Call
Heating Degree Days (HDD)
Contract period: November 1, 1999 – March 31, 2000
Weather Station: Chicago/ O'Hare Airport
Strike: 5000
Dollars per unit: $10,000
Maximal payout: $2,000,000

Beginning on November 1, 1999 we repeat the following process every day: take the maximal weather recorded at the weather station and the minimal weather. Take the average of the two numbers. This determines the "weather" for that day.

For each day *i*, the station releases *Tmax(i)* and *Tmin(i)* the maximal and minimal temperatures recorded on that day. We define *W(i)*, the weather for day *i* as
W(i) = (Tmax(i) + Tmin(i))/2

The next step involves looking at the difference between the weather of the day and 65 degrees. Since this is an HDD option, we only consider cold days in which the weather was below 65 degrees. These are days in which people would have to turn their heaters on. Thus *HDD(i)*, the heating degree days measured on that day would be
HDD(i) = max{65-W(i), 0}
Similarly, options trade on cooling degree days CDD. Therefore *CDD(i)*, the cooling degrees generated on day *i* would be
CDD(i) = max{W(i)-65, 0}

The third step involves summing the HDD(i) for all the days in our contract period, including the first and last days, November 1, 1999 and March 31, 2000 respectively.

Total = Sum(i=November 1, 1999 to March 31, 2000) [HDD(i)]

On the day following the expiration of the option, that is April 1, 2000 we can compute the payout

*Payout = min(Dollars per unit * max(Total-Strike, 0), Maximal payout)*

In our example,

*Payout = min($10,000 * max(Total-5000, 0), $2,000,000)*

The party who buys this option will be paid if the winter in Chicago will be severe. In that case, the winter will be cold, a lot of HDD's will be generated and *Total* would be large. Thus call options pay out in extreme weather conditions and put options pay out when the weather is mild.

Actually, the payout of this option shows a greater similarity to a traditional call spread rather than to a call option. Note that our option can not be in the money by more than 200 HDD's.

In addition to put and call options, there have been some trading in digital options, barrier structures and even compound options.

Motivation

There are many institutions whose profit and loss are severely impacted by the weather. For example, farmers whose crop will be ruined if the weather is too warm, may desire these options as a hedge. Other potential clients are clothing manufacturers and retailers whose livelihood depends on cold winters. Tour operators, hotels and many types of seasonal businesses could also benefit. The US Department of Commerce estimates that weather impacts businesses representing $1 trillion of the $9 trillion US gross domestic product.

In addition, small institutions on the buy side may be wary of purchasing options on traded commodities. An often-expressed fear is that the big banks on the sell side will trade in the spot market in such a way as to cause some barrier options to knock out. However, even the largest sell side dealer can not influence the weather.

Pricing

Black Scholes

The Black Scholes pricing methodology is based on continuous hedging. This method works well when pricing options on currencies, stocks, commodities or other fungible assets which can be traded in the spot market. The difficulty with the weather derivatives is that the underlying is not traded. "You can not buy a sunny day" goes the old saying.

Burn Analysis

Another approach is practiced in the insurance industry. One may ask the question: "what would we have paid out had we sold a similar put option every year in the past fifty years?"

The method proceeds as follows:

1) Collect the historical weather data.
2) Convert to degree days (HDD or CDD).
3) Make some corrections.
4) For every year in the past, determine what the option would have paid out.
5) Find the average of these pay out amounts.
6) Discount back to the settlement date.

This method is known as "burn analysis". Much of the difficult work is in stages 1 and 3.

Collecting the historical data may be somewhat difficult. While there are web sites with downloadable historical weather information for the USA, obtaining historical weather data for the United Kingdom is quite costly. Even when the data is available, there are missing data, gaps and errors. The historical data must be "scrubbed" before it is used for pricing.

The correction step is also challenging. Here are some issues:

1) The period in question is a leap year. Thus there are more days in the period November 1, 1999 – March 31, 2000 than there are in the corresponding period (November 1, 1998 – March 31, 1999) the year before. This is due to the existence of February 29, 2000, a date that did not exist in 1999.

2) The weather station may have had to be moved due to construction. Alternatively, it may have been in the sun and now it is in the shade etc.

3) How many years of historical data should one consider? Should you look at ten years? Twenty? Or even more?

4) Many cities exhibit the "urban island effect". Due to heavy industrial activity, construction and pollution, the weather gradually becomes warmer in that area. In fact, in some urban centers, it is possible to detect warming trends in the weather. These trends must be accounted for when pricing the option.

5) There are extreme weather patterns that occur in some years. Most notable are the El Nino and La Nina. Basically, the El Nino is the warming of the water in the eastern and central equatorial Pacific Ocean. La Nina is the opposite effect when the same areas of the Pacific are cooler than average. The pricing of an option in a year that is forecast to be El Nino is different than in a year that is not.

In any case, it is possible to resolve these problems in a reasonable manner. A weather trend may be corrected for by observing the average HDD's for the past ten years and comparing it with the corresponding trailing ten year average of the HDD's for each of the years in the past. For example, assume that

- For the year 1988, in the period November 1, 1988 to March 1, 1989 the HDD count was 5050.
- The average of the HDD counts for the ten years 1989 to 1998 was 5000.
- The average of the HDD counts for the ten years preceding and including 1988, which are 1979 to 1988, was 5020.

When we compare the average HDD counts in that city, we find that the HDD's have dropped from 5020 to 5000. This is consistent with a warming of the weather. In this case, it may be reasonable to shift the data relating to 1988. A linear shift would be

Shifted HDD = Observed HDD + last average – previous average = 5050 + (5000-5020) = 5030

It is also possible to find reasonable corrections for the other effects.

Most market participants use some sort of burn analysis in computing the fair value of the option. These "degree day based models" are simple to construct. All that is required is a good source of historical data.

A Flaw

There is a serious flaw in the degree day based models such as the burn analysis. This flaw manifests itself most profoundly in periods when temperatures hover around 65 degrees. In most areas of the US, these are typically the fall and autumn seasons, so called the "shoulder months".

To observe this flaw, consider a simple contrived hypothetical example. Consider an option whose period is three days: February 3, 4 and 5, 2000.

We observe the historical weather in City A:

	Feb 3	Feb 4	Feb 5
1996	64	64	64
1997	66	66	66
1998	64	64	64
1999	67	67	67

We also observe the historical weather in City B:

	Feb 3	Feb 4	Feb 5
1996	64	64	64
1997	96	96	96

1998	64	64	64
1999	97	97	97

Obviously these two cities exhibit totally different weather patterns. We would not want to sell a weather derivative on City A for the same price as a weather derivative on City B.

Note, however, that any degree day bases model (including the burn analysis described above) would not be able to distinguish between these two cities.

The HDD's generated are exactly the same for both cities:

1996	3
1997	0
1998	3
1999	0

Any degree day based model can not differentiate between City A and City B. The problem is due to the cutoff at 65 degrees.

Another Flaw

What is the level of a zero cost swap? Under burn rate analysis, the answer to this question depends on the maximal payout assumption. This seems highly non-intuitive, as we would expect that the strike of a zero cost collar would be the same no regardless of the maximal payout amount.

For example, consider a swap in which we are long HDD in Chicago. The period is November 1, 1999 to March 31, 2000. Assume that there is a $10,000 payment per HDD and that the maximal payout is $10,000,000.

Taking the last ten years of data (1989-1998), without trending or adjusting for leap years, we note that the average HDD level is 5018.75. A swap with a level of 5018.75 would indeed be a zero-cost swap.

On the other hand, assume that the maximal amount that can be paid out (either way) under the swap is only $1,700,000. In this case, the burn analysis gives a totally different result. A swap with a level of 5018.75 would actually show an average payout of about ($202,000). That's right, the average payout of this swap would be approximately negative $202,000!

If we restrict the maximal payout, then in some of the ten years of history we would get the maximal payout of $1,700,000 and in some we would pay it. The average of these payouts will not, in general, be zero. In some cases (as in our example) it would be very different from zero. To make the average payout zero, we would have to change the swap's level to 4945.

In summary, if the maximal payout of the swap is $10,000,000 then the level of a zero cost swap is 5018.75. On the other hand, if the maximal payout is $1,700,000 then the level of the zero cost swap becomes 4945.

The problem does not disappear even if we used more data. Using thirty years of historical data gives very similar results.

This very un-intuitive result stems from the fact that we are using a burn rate analysis and that the options' payout is bounded.

Temperature Based Models

More sophisticated weather derivative models are based on modeling the weather directly. These models do not incur the flaws mentioned above.

Such models have the following steps:

1) Collect the historical weather data.
2) Make some corrections.
3) Create a statistical model of the weather.
4) Simulate possible weather patterns in the future.
5) For each weather pattern, calculate the payout of the option.
7) Find the average of these pay out amounts.
8) Discount back to the settlement date.

The fundamental difference between the two approaches is that we are building a model for the weather, not the degree days.

The simulation done in step 4 could be performed using a Monte Carlo algorithm. Such algorithms generate random numbers. These random numbers are then used to simulate the behavior of the phenomena we are trying to model.

It is possible to price foreign exchange options using the Monte Carlo algorithm. Most market participants agree that foreign exchange fluctuates according to a random walk described by a geometric Brownian motion:

The stochastic process is described by the differential equation:

$$dP = (r-q)P\,dt + \sigma P\,dz$$

Here
P is the price of the security (or the foreign exchange rate)
dP is the instantaneous change to the price P
dt is an infinitesimally small unit of time
r is the domestic interest rate of the payout currency

q is the foreign interest rate
σ is the annualized volatility of the exchange rate
dz is a Weiner process based on a normal distribution with a mean of zero and a standard deviation of 1:

It is possible, and not too difficult, to show that an algorithm which involves running many simulations of this Monte Carlo algorithm, taking the final exchange rate on the expiration of the option, computing the payout of the option for each of the simulations, averaging these payouts and discounting the average back to the settlement date will give precisely the same results as the Black Scholes formula.

Can we use the same model for temperature?

Unfortunately not. It is possible for exchange rates or stock prices to fluctuate sharply over time. For example, many stocks have doubled their value within one year. However, it seems unlikely that the temperature next year will be double of what it was this year. We therefore choose a different model for the weather. To model weather, we decided to use mean reverting models.

Mean Reverting Models

Mean reverting models have been used extensively to model interest rates. In the US, where interest rates are approximately 5%, it is unlikely that rates will be 50%.

There are many different models of interest rates. For an excellent review of the different models, see the chapter by Kerry Back in my book "Option Embedded Bonds", Irwin Professional Publishing (ISBN 0-7863-0818-4).

To illustrate a mean reverting model, consider the "simple Gaussian model". The differential equation describing the model is given by:

$$dr = a*(b-r)\, dt + \sigma\, dz$$

Here
r is the continuously compounded instantaneous interest rate
dr is the instantaneous change in r

dt is an infinitesimally small unit of time
b is the mean interest rate
a is the speed of mean reversion.
σ is the volatility
dz is a Weiner process based on a normal distribution with a mean of zero and a standard deviation of 1:

This is an example of a simple "mean reverting" model. Intuitively, *r*, the instantaneous interest rate changes by an amount equal to *dr*. In this model, it is assumed that interest rates will converge to some "long term mean" *b*. If *r* is greater than *b* then the contribution *(b-r)* s negative. This will tend to pull interest rates to a lower level. Similarly, if *r* is less than *b*, then *(b-r)* is positive which will tend to pull interest rate higher.

The term *(b-r)* is the "pull to the mean". It is multiplied by *a*, the "speed of mean reversion" and this is added to the term *dr*.

In addition, there is a random component to the short term interest rate. The random component is represented by the term $\sigma\, dz$.

dz is a Weiner process that is based on a normally distributed random variable which has a mean of zero and a standard deviation of one. Thus the random component may be positive or negative.

Mean reverting models, similar to the "simple Gaussian", are used to price interest rate options, such as caps, floors and swaptions. There is an active and liquid market for interest rate derivatives based on the Libor (London Interbank Offered Rate). Indeed, the most liquid at the money caps will be offered by various dealers at prices that are within 0.1 basis points of each other.

The main difficulty lies in determining the various parameters: *a,b* and σ. Obviously, once the parameters are known, it is possible to compute the prices of various derivative instruments. Calibration is the reverse process in which the market prices of the liquid derivative instruments are used to determine the parameters of the model. We are trying to determine the set of model parameters that would result in prices that are as close as possible to the market prices on a large variety of instruments.

Intuitively, calibration answers the question:
"What is the set of model parameters that would result in the model matching the observed market prices or as coming as close to them as possible"

Model calibration is quite computationally intensive and typically requires high dimensional non-linear optimization. Of course, it relies on the availability of market prices for the liquid instruments.

Some questions

It is natural to assume a mean reverting model for the weather. Just like interest rates, it is unlikely that the weather next year will be ten times higher than the weather this year.

Before embarking on designing a mean reverting model for the weather, though, we need to ascertain if such a model describes the weather reasonably well.

One question to ask is: "Is the weather distributed normally around its mean?"

In Figure 1 we plot the Cumulative Distribution Function (CDF) of the weather in a particular weather station (Chicago). We compare the CDF to the one generated by a normal distribution. In most cases, the observed CDF of the weather closely matches the CDF of a normal distribution. There is several statistical test to quantify the "goodness of the fit". One such test is the Kolmogrov-Smirnov (KS) test. The KS test measures the maximal distance between the hypothetical CDF and the observed one. This is termed the "ks score". Based on the "ks score", the probability that the distribution is not normal is displayed. Obviously, high ks scores would imply a high probability that the distribution is not normal.

For Chicago, in 42 days (out of 366), one is able to reject the hypothesis that the weather is distributed normally. In the remaining 324 days, we can not reject the hypothesis that the weather is distributed normally. Since the normal distribution assumption holds well for about 88% of the cases, we have decided to use it.

These results are quite typical. In extensive tests over many different weather stations, we've noticed that we can reject the normal distribution in about 10-20% of the days.

Another question that we may ask concerns the degree of the model.
When you are modeling tomorrow's temperature, you should obviously use the temperature from today. However, should you also use the temperature from yesterday as well as the day before yesterday? What about 50 days ago?

We can write this as an equation:

$$W(T+1) = F(W(T), W(T-1), \ldots , W(T-n))$$

Where, $W(T+1)$ is the weather tomorrow, $W(T)$ is the weather today, $W(T-n)$ is the weather n days ago and F is some unspecified function.

How many days from the past should we use. What should n be? Models in which n is zero, are called memory-less. In other words, they exhibit a Markov property.

This concerns the auto-correlation between the temperatures. In Figure 2, we plot the auto-correlation for a particular weather station (Chicago). Naturally, the weather on each day has a perfect correlation with itself. Thus for a lag of zero, the correlation is 1. If we allow a one day lag, the correlation is quite high (about 0.7). Note that the correlation decreases as we increase the lag.

If the correlation between $W(T+1)$ and $W(T-n)$ is exactly zero, then no new information added by using that day. It is almost impossible to see a correlation of exactly zero between any two series. However, if the correlation is small enough and is close to zero, then very little information is added by using that day.

Figure 2 shows that there is some correlation even when n is 25. In general, the higher the number n, the more precise our model. On the other hand, the higher the number n, the more parameters that will have to be estimated. This will cause the calibration process to be more complicated. That is, we would have to determine exactly how $W(T-n)$ impacts on $W(T+1)$. As n increases and the correlation decreases, the effect becomes subtle and more difficult to determine.

We have only a limited amount of historical weather information and the observed weather in the past is subject to random fluctuations. This causes "parameter estimation errors". The relative importance of these errors grows with n.

When designing a weather model, what n should one choose? Because of the difficulties caused by the parameter estimation errors, we have decided to use $n=0$. Thus our models assume that the weather is a Markov process. Obviously, this is only an approximation. However, given the realities of the market place and the available data, we felt that this was the best choice.

A Model for Weather

Our model is similar to the models used in interest rate derivatives. However, there are a few caveats:

1) The weather changes with the season. Hence we allow the mean of the weather to vary. The parameter representing the mean, b, is replaced with $b(i)$, which represents the mean for day number i.
2) Similarly, the volatility may depend on the day in question. In many cities (e.g. Chicago) the weather is more volatile in the winter than in the summer. The volatility parameter σ is changed to $\sigma(i)$, the volatility for a particular day.
3) By the same token, we allow the mean reversion rate to vary. The parameter a, which represents the mean reversion rate is allowed to change over time. The mean reversion rate for day i is represented by $a(i)$.
4) There is a natural seasonal effect in weather. Assume that it is now spring and that the temperature today is exactly equal to its long term mean. We may well expect that the temperature tomorrow will be slightly warmer than it is today. In other words, there is a natural "drift" to the weather.

The most important difference between interest rate derivative models and models for weather derivatives is the calibration process.

- Interest rate derivative models are calibrated to the market prices of liquid instruments. This calibration process was described above.
- Weather derivative models are calibrated to past data.

As yet, an active and liquid market does not yet exist for weather derivatives. On the other hand, we have a wealth of historical weather data.

The calibration process asks:
"What is the set of model parameters that would have the highest probability of generating the past weather patterns?"

This is essentially a "maximum likelihood" question. Assume that the observed data is the result of a stochastic process. We are determining the parameters for which the probability of having generated the observed data is maximal.

For example, consider that you flip a coin 1000 times. It comes out 900 heads and 100 tails. It could be that this is a fair coin and that this is an unlikely sequence of coin flips. On the other hand, it could be that the coin is not even and intrinsically has a much higher chance of showing heads than tails. The maximum likelihood technique would tend to choose the second explanation.

To summarize, the weather derivative model works as follows:

1) It calibrates the model to the observed past data using a maximum likelihood technique.
2) Once the model parameters are determined, weather sequences are generated using a Monte Carlo process.
3) The random sequences drive a mean reverting model, similar to models used to price interest rate derivatives.
4) Many sequences are generated. Each sequence represents a possible future weather pattern. For each weather sequence, the payout of the option is determined.
5) The average payout of the option under the various scenarios is deemed to be the expected payout of the option.
6) Taking the present value of the expected payout gives us a fair value price.

Hedging of Weather Derivatives

Option traders, including equity and foreign exchange option traders, utilize a technique known as "delta hedging". Delta hedging entails computing the sensitivity of an option's price to a change in the underlying and buying or selling an appropriate amount of underlying instruments. For example, suppose a dealer sold an at the money call option with a delta of 0.50. This means that for every $1 rise in the price of the underlying stock, the option's price will rise by $0.50. A dealer who sells such a call option immediately buys 0.50 units of stock. If the stock rises by $1, the dealer will lose $0.50 on his option position but will gain $0.50 because he is long shares.

The difficulty in hedging weather derivatives lies in the fact that one can not purchase or sell the underlying instrument. As the saying goes "You can't buy a sunny day".

Weather derivative dealers hedge themselves using a variety of techniques, including:

1) Limiting the potential loss on an option. All weather derivatives have a cap on the maximal payout. This is quite different than traditional options (e.g. on equities or foreign exchange) that typically do not have caps on the maximal payout amounts.
2) Running a well diversified, balanced book. Dealers try to buy and sell many weather derivatives that are based on both HDD and CDD in many different cities. Even if the weather deviates from normal by a lot in one part of the country and causes a loss to the dealer, it is highly unlikely that the dealer will lose on all open option positions. Note that dealers have to assume some correlation (or lack of correlation) between the various weather stations. Such assumptions can be tested on past data. In this fashion, the weather derivative dealer is essentially working like an insurance company. An insurance company sells many car insurance policies and buys some re-insurance products. Unfortunately, a well balanced book is difficult to create a small weather derivative business.
3) Selling options that are long term. A typical option sums up all the degree days between November 1 and March 31 of the following year. Even if the weather is unseasonably warm on a particular day, it is unlikely to stay extremely warm during the entire five months. A dealer would be hard pressed to sell a weather derivative that is settled based on the weather on a single day. The summation effect reduces the variability of the pay out.

One hedging technique that is available is to hedge with zero cost swaps.

Assume that a dealer sells a 5000 HDD call option on Chicago. The option runs from November 1 to March 31. It pays $10,000 per HDD with a maximal payout of $2,000,000. Here are some modeling results:

ATM DD	Sold 5000 Call	P&L on Call	Buy 5016 call	Sell 5016 Put	price of swap	P&L of swap
4,966	$709,649	$ 113,104	$668,830	$869,192	$ (200,362)	$ (200,216)
5,016	$822,753		$782,386	$782,532	$ (146)	
5,066	$910,081	$ (87,329)	$889,733	$725,206	$ 164,527	$ 164,673

Assume that the zero cost swap strike price is 5,016.

The price of the 5,000 call option which was sold by the dealer is $822,753.

Assume that several days have past and that the forecasters change their prediction. Based on new information they predict that the winter will be slightly milder than usual. That would mean that the strike of the zero cost swap would move downwards. Assume that it would change to 4,966.

If the zero cost swap strike was to move by 50 HDD to 4,966 the price of the 5,000 call would decline to $709,649 and the dealer would show a profit of $113,104. On the other hand if the zero cost swap strike was to move by 50 HDD to 5,066 the dealer would show a loss of $87,329.

Let's consider the 5,016 swap. The swap consists of a long position in a 5,016 call option and a short position in a 5,016 put option. Both options have a payout of $10,000 per HDD with a maximal payout of $2,000,000

When the at the money swap strike is 5,016, the price of the call almost exactly offsets the price of the put and the cost of the swap is zero (a zero cost swap).

If the zero cost swap strike was to move to 4,966 the price of the 5,016 swap would decrease by $200,216. On the other hand, if the zero cost strike price would increase to 5,066 the price of the 5,016 swap would increase by $164,673.

As the dealer does not know what the future will bring, he should hedge himself by entering into a fractional amount of a swap. The amount of swap entered in this example would be:

($113,104- (-$87,329)) / ($164,673 – (-200,216)) = 0.549

Note that the "delta" of the call option is 0.549. We expect the delta of a slightly in the money call option to be slightly greater than 0.5. Also note that the profit on one side $113,104 does not exactly match the loss on the other side $87,329. This is due to the gamma effect. The delta of the option changes as the strike of the at the money swap increases or decreases.

Therefore, our dealer could enter into 0.549 units of swap for every call option sold. In reality, the dealer would probably adjust the payout amounts per HDD and the maximal payout amount.

Assume that the 5,016 options entered into as the swap position were to be changes so that the payout is $5490 per degree day and the maximal payout is $1,098,000.

We now repeat the calculations as above:

ATM DD	Buy 5016 call	Sell 5016 Put	price of swap	P&L of swap
4966	$ 376,427	$ 479,616	$ (103,189)	$ (103,376)
5016	$ 431,435	$ 431,248	$ 187	
5066	$ 487,091	$ 399,120	$ 87,971	$ 87,784

We note that the profit and loss on the swap almost matches the profit and loss on the option exactly.

In a traditional delta hedging approach, the dealer would constantly adjust his delta hedge ratio and increase or decrease his exposure to the swap. This may be very costly to do in the weather derivatives market as the instruments are not liquid. Therefore, if dealers use delta hedging at all, they would hedge only at the beginning of the trade and will typically not re-balance at all.

Summary

To summarize, the weather derivative product is rapidly developing. While its origins as an insurance policy explain the usage of techniques such as “burn analysis”, they are quite inadequate to price these options. We have developed techniques, models and tools that are based on interest rate pricing analytics. It is only a matter of time until such models are adapted by the industry.

Appendix: Figures

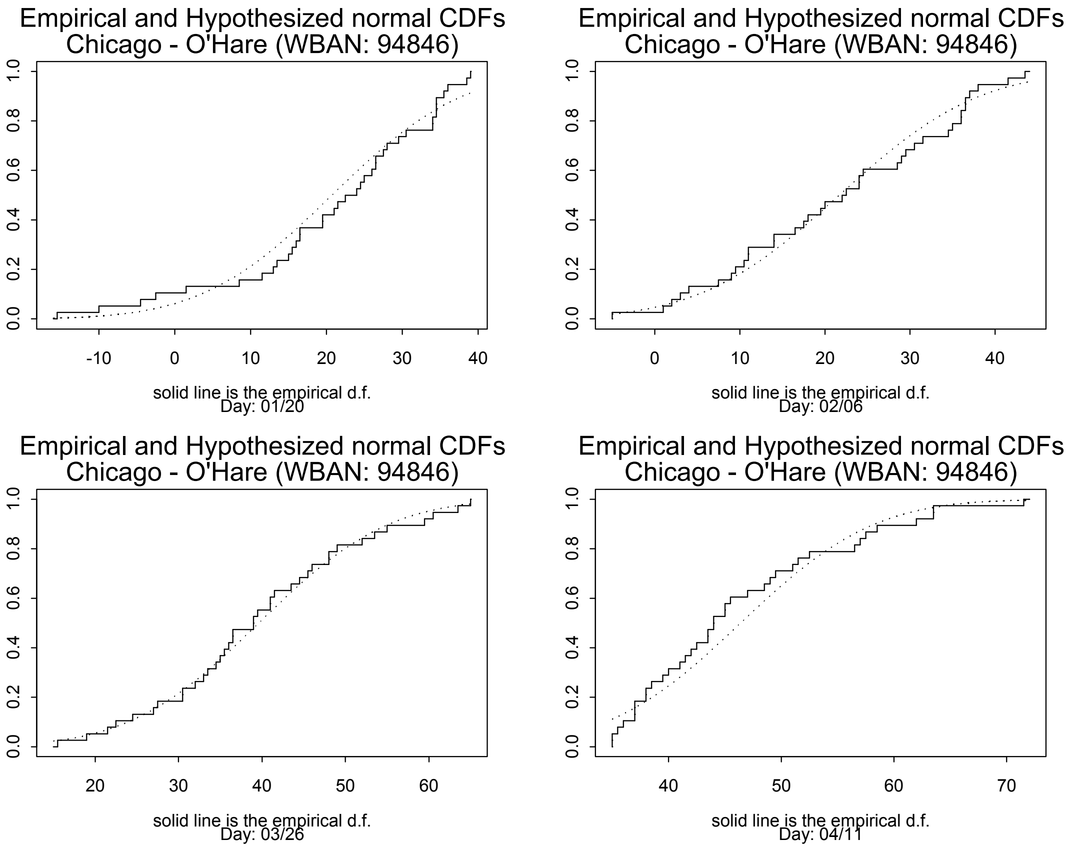

Figure 1: Several days and their Cumulative Distribution Functions (CDF) as compared with the normal distribution. Note the goodness of the fit.

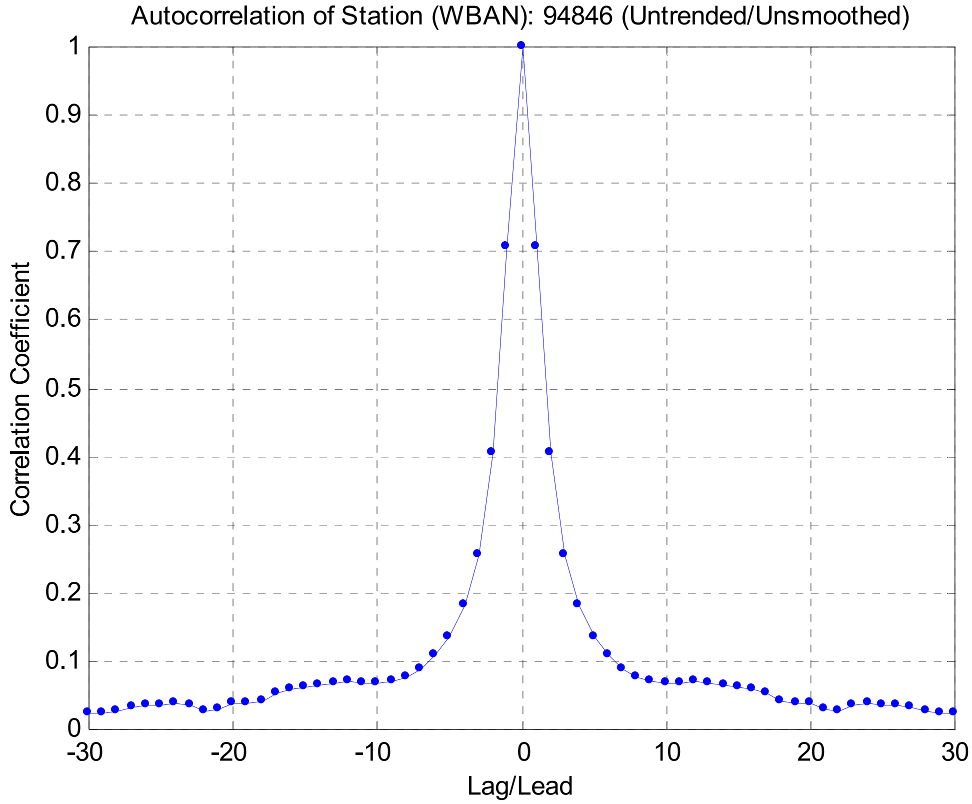

Figure 2: Autocorrelation of the temperatures.