From Log Contracts to the CBOE's "New VIX" index

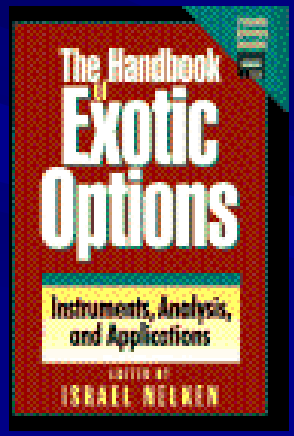
How a completely theoretical idea turned into a very practical tool

With many thanks to Krag Gregory and Venkatesh Balasubramanian at Goldman Sachs

Izzy Nelken
Super Computer Consulting, Inc.
3943 Bordeaux Drive
Northbrook, IL 60062
(847) 562-0600
www.supercc.com
www.optionsprofessor.com
Izzy@supercc.com

A little history

Nine years ago, in 1995, I was asked to edit “The Handbook of Exotic Options”



Log contracts

Professor Anthony Neuberger of the London Business School proposed to write a chapter on Log Contracts extending a 1994 paper by himself.

Stable “Greeks”

- As Professor Neuberger pointed out, one of the features of a contract on the Log of a stock would be that it has “stable Greeks”
- Contour plots of the Delta and Gamma of a normal call option against time & stock price results in non-linear curves
- Similar plots for a Log call option result in straight lines

Pure bets on Volatility

- “The log contract provides a much easier and more reliable way of betting on volatility”

In 1995

- In 1995, this was a purely theoretical result
- To paraphrase Yosemite Sam

"Who Cares"

Volatility and Variance Swaps

In the April 1998 issue, Derivatives Strategy mentions:

two new products released in January: volatility futures from the Deutsche Terminborse and volatility swaps from Salomon Smith Barney.

Pre packaged Volatility Plays: Derivatives Strategy, April 1998

The Market Matures

Variance swaps took off as a product in the aftermath of the LTCM meltdown in late 1998 when implied stock index volatility levels rose to unprecedented levels. Hedge funds took advantage of this by paying variance in swaps (selling the realized volatility at high implied levels). The key to their willingness to pay on a variance swap rather than sell options was that a variance swap is a pure play on realized volatility - no labor-intensive delta hedging or other path dependency is involved. Dealers were happy to buy vega at these high levels because they were structurally short vega (in the aggregate) through sales of guaranteed equity-linked investments to retail investors and were getting badly hurt by high implied volatility levels.

Replication of Quadratic Variation-based Payoffs by Jim Gatheral, Merrill Lynch

A seminal paper

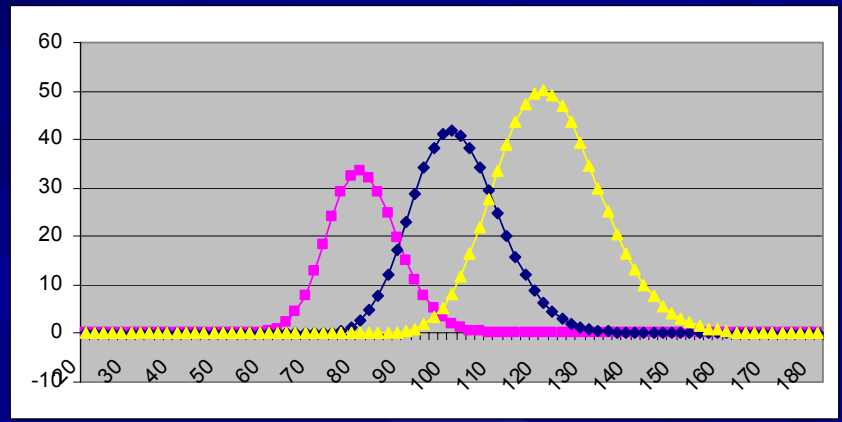
In March 1999, Emanuel Derman and his colleagues published: “More Than You Ever Wanted To Know About Volatility Swaps But Less Than Can Be Said”

"More Than You Ever Wanted to Know About Volatility Swaps" by Kresimir Demeterfi, Emanuel Derman, Michael Kamal and Joseph Zou, Goldman Sachs Quantitative Strategies Research Notes, March 1999.

“Variance Vega”

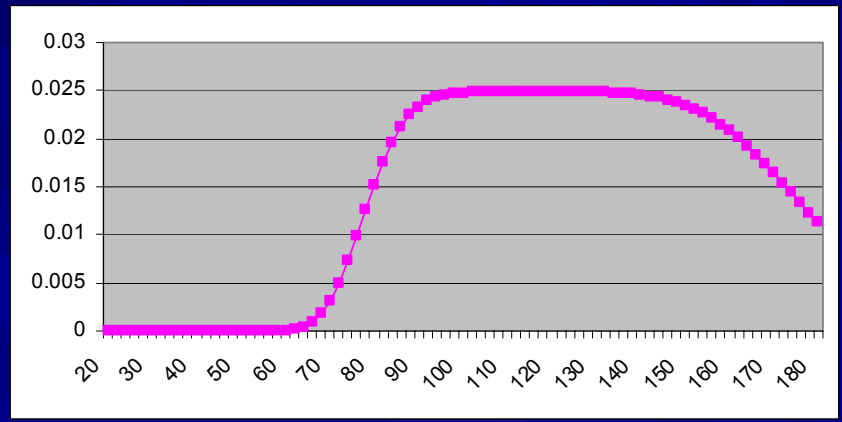
This is a plot of the “Variance Vega” of options with differing strikes

Variance vega – how the price of the option changes with a small change to the square of the implied volatility



A weird portfolio, indeed

- Buy $1/k^2$ units of options with strike k
- The portfolio's sensitivity to the variance is insensitive to the stock price (across a wide range of stock prices)
- Actually, we use $\Delta k/k^2$ units for normalization purposes.



The Log contract - revisited

Derman's paper showed that

by rehedging the position in log contracts, you have, in effect, been the owner of a position in a variance swap with fair strike $K_{var} = \sigma_I^2$ *and face value $1*

The first energy volatility swap

- Fast forward to Nov. 7, 2003

  KOCH SUPPLY & TRADING COMPLETES ITS FIRST ENERGY VOLATILITY SWAP

  “The transaction allowed us to make a clean, relative-value bet on the future realized volatility of WTI versus heating oil,” says John Arnold, principal of Centaurus Energy. “The volatility swap lowers our transaction costs as we avoid having to gamma trade futures in order to stay delta-neutral.”

Proliferation of exotics

- In the 1990's we had:

  Exotic options on vanilla underlying instruments

  (Chooser, Compound, Lookback, Double Barrier, One Touch and more)

- The 2000's are characterized by:

  Vanilla options on Exotic underlying instruments

  (Credit Default Swaps, Weather Derivatives, Volatility Contracts)

Realized Volatility is NOT Implied Volatility

- Volatility swaps trade the realized volatility of the underlying instrument (standard deviation of the returns)
- The VIX index (old and new) are instruments related to the implied volatility of options

The New VIX index

- Uses a formula similar to the one described by Derman et al. for Variance Swaps

The Log contract

- Which was first discussed in 1994 as a purely theoretical construct
- ...turns out in 2004 to be extremely important for the construction of the CBOE VIX index.

- From pure theory to a practical application

Volatility Dispersion

- A key question: how is the volatility of an index related to the volatility of its components?

IN1

Slide 18

IN1 I spoke about this last year at the CBOE's 19'th Risk Management Conference. They've invited me again this year but I was already committed to a seminarr at the Bank of England

IZ NL, 3/16/2004

The Portfolio Equation

$$Var_p = \sum_{i=1}^{n} \sigma_i^2 w_i^2 + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} \rho_{i,j} \sigma_i \sigma_j w_i w_j$$

$$\sigma_p = \sqrt{Var_p}$$

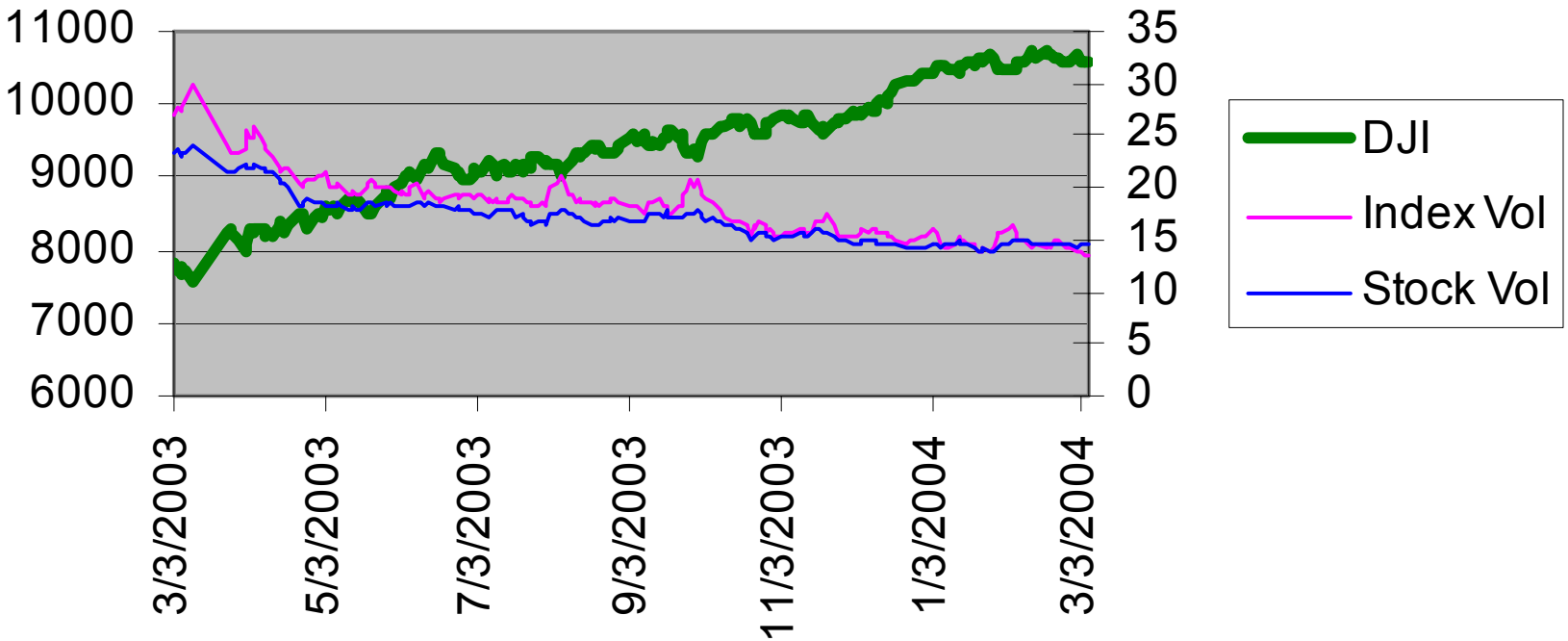
Portfolio Equation - II

σ = standard deviations of returns
ρ = correlation of returns
w_i = weights for each security

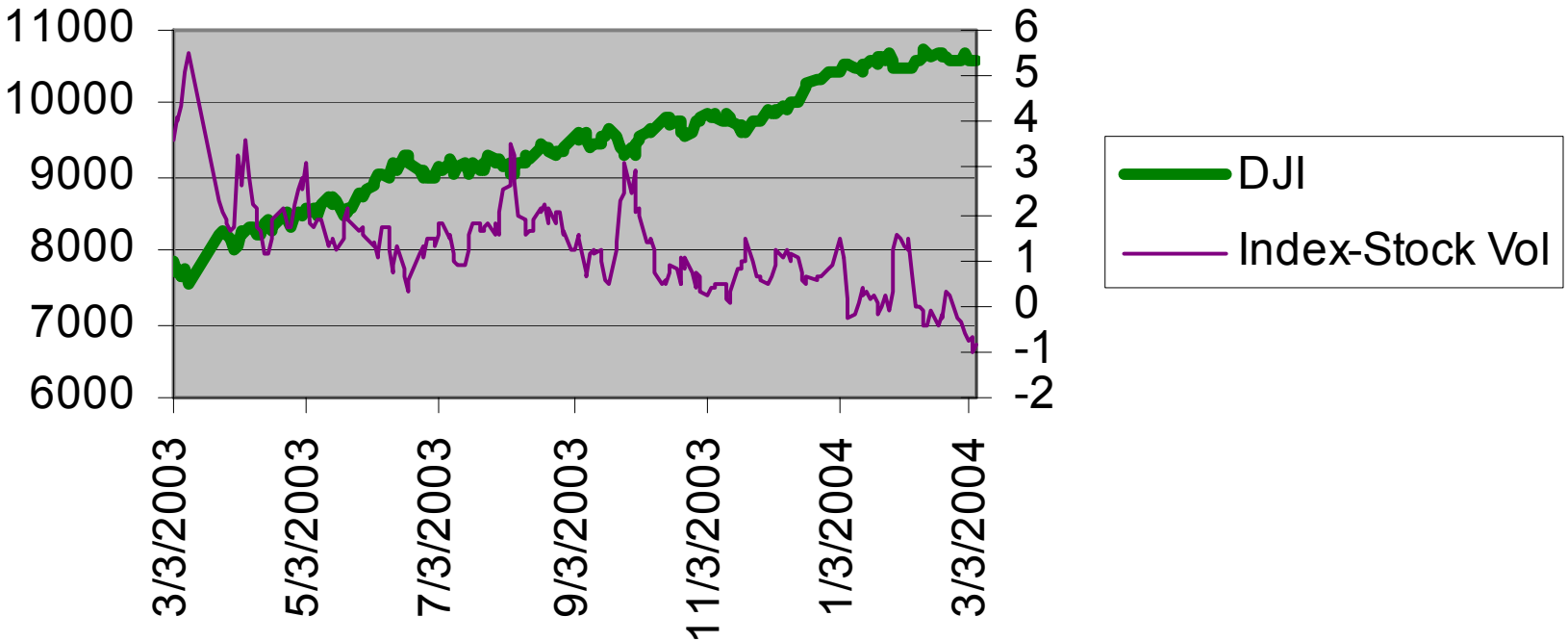
Correlation and Volatility

- We can take the historical correlation of the stocks
- Volatility of the index and the components
  - Traditionally, use the ATM implied volatility
  - New idea, use the New VIX methodology to come up with a new volatility measure

ATM Volatility



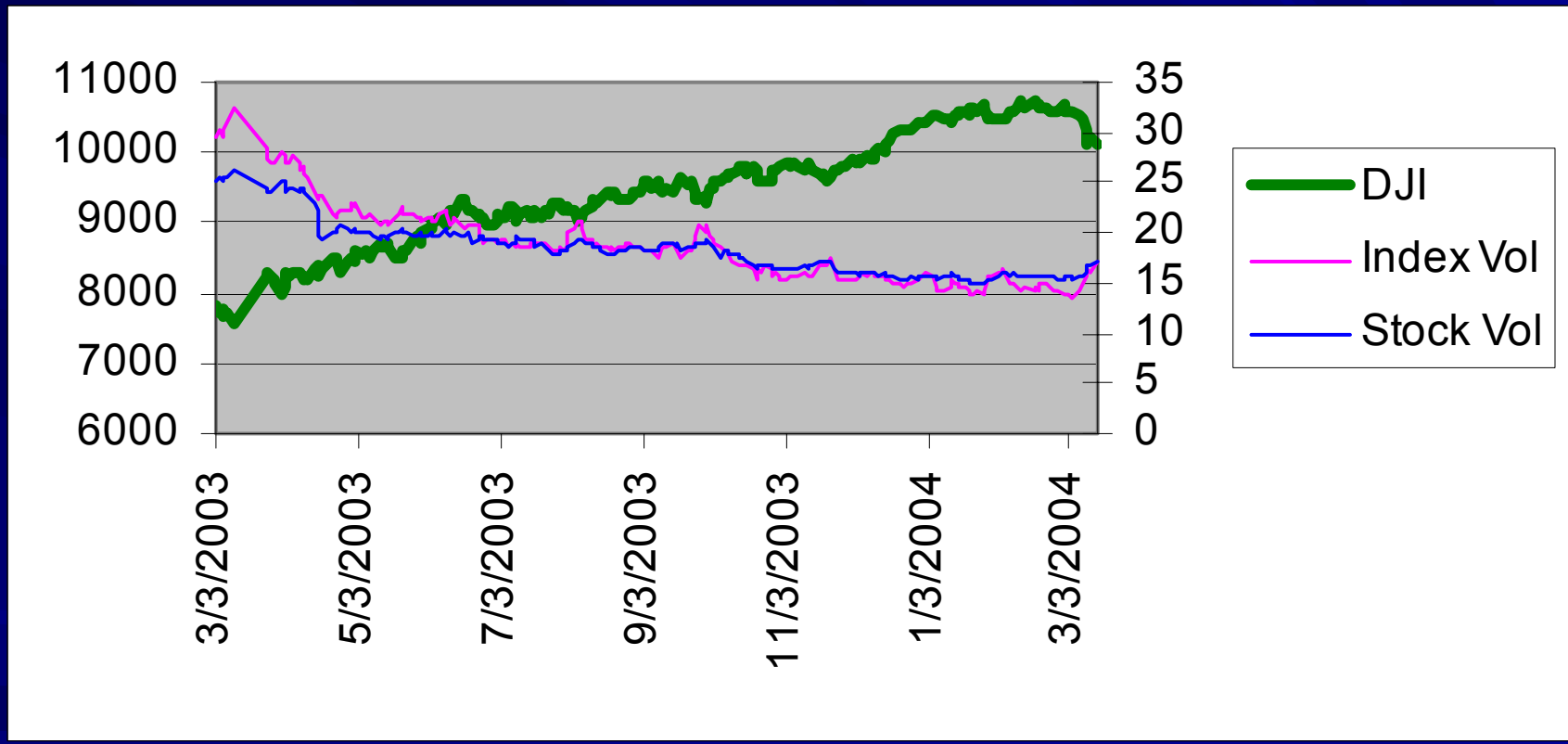
ATM Volatility Difference



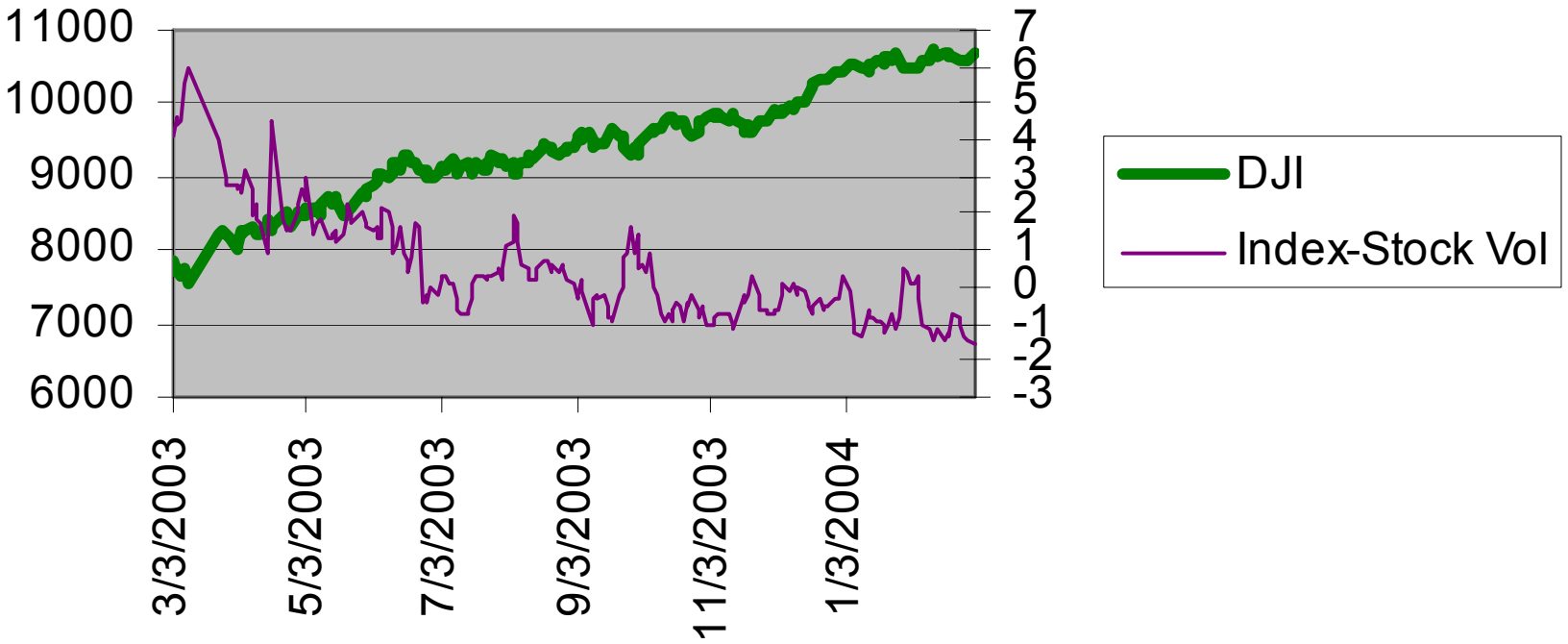
Why is there a difference?

- We compute the theoretical volatility of the index based on the volatility of the stocks
- The assumes the historical correlation
- The market has its own view on correlation
- When volatility is high, people are nervous (afraid of a crash) and the “implied correlation” is higher than the historical correlation

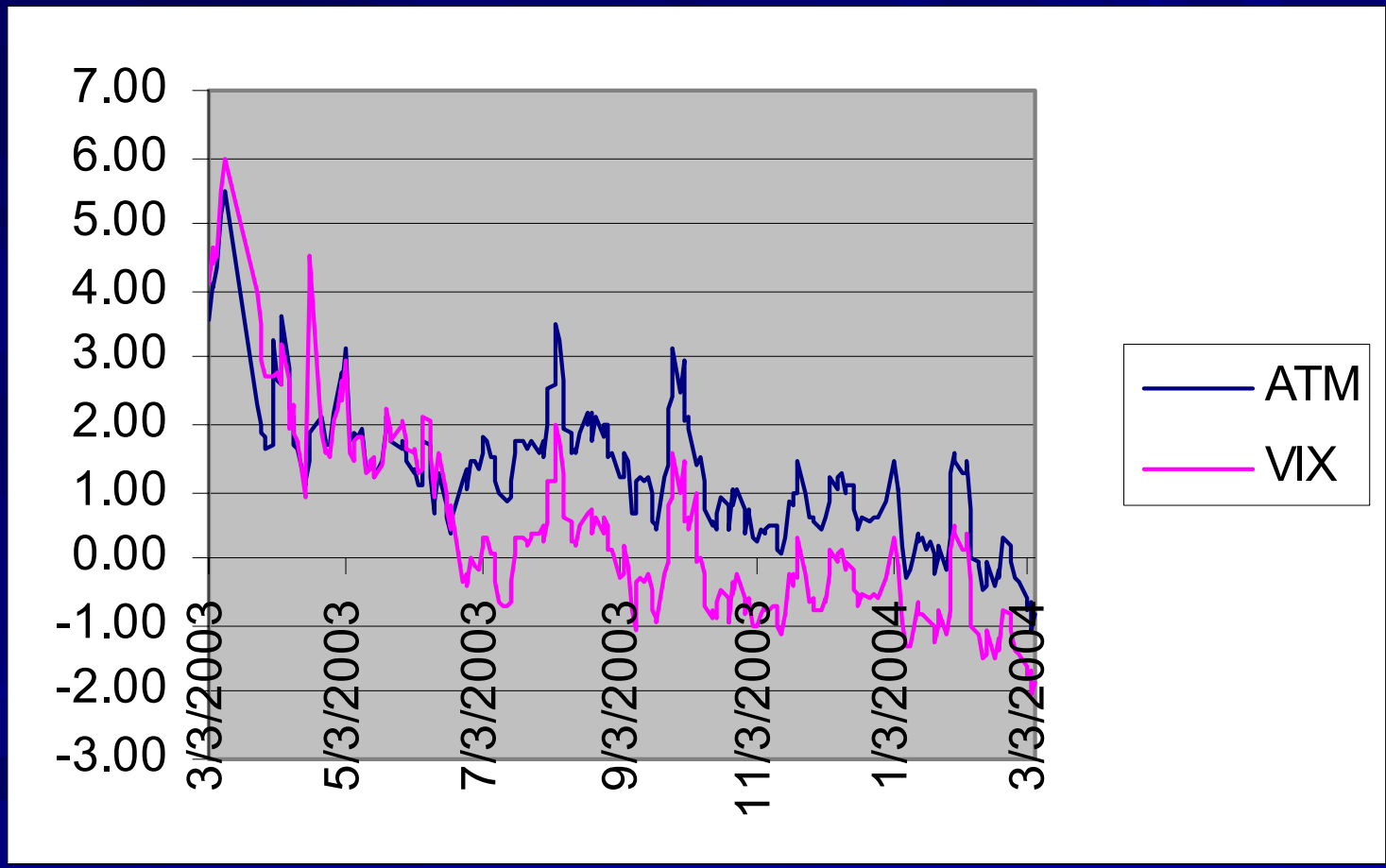
VIX Volatility



VIX Volatility Difference



Combined Volatility Differences



Comparison

The volatility difference done using the VIX methodology is much less volatile than the volatility difference done using the ATM methodology

IN2

IN2 Vol of the difference is 200% vs. 338%

IZ NL, 3/19/2004

Summary

- Currently, volatility dispersion trades are complex to execute in the open market:
  - ATM options require constant re-balancing
  - The new VIX technology is not available for single stocks
- In the future, it may be possible to develop new VIX indices for single stocks
- This may open the door for volatility dispersion trading